Grip Meter

Mid- Semester Report

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Fall 2011

Stroke is a leading cause of disability in adults. It is caused by a disturbance of blood supply to brain, leading to loss of functions including in some cases hand motor skills. A grip meter measures the grip strength of individual and is used to gage the improvement of a patient in their grip over time. Current grip meters are expensive and do not cover the range of force our client would like. Our device needs to measure from 0 to 9.07 kg (0 to 20 lbs) with .454 kg (1 lb) increments. In our design, a strain gauge will measure the force, which will be displayed on LCD screen. It will use a similar grip system as current devices.

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1.0 Introduction

1.1 Background

A stroke is caused by the disruption of blood flow to part of the brain, ultimately resulting in a loss of brain function [1]. There are two types of strokes: ischemic and hemorrhagic. An ischemic stroke occurs due to blood clot formation, either within the brain or somewhere else, which then travels to the brain. A hemorrhagic stroke occurs when a blood vessel within the brain bursts and leaks into other parts of the brain. Both types have serious consequences that can result in a plethora of different effects, depending on the location of the stroke. The sufferer of a stroke may have trouble hearing and seeing, whereas another person may not be able to move their left arm; it all depends on the individual [1]. Signs of stroke can include a constant headache, change in hearing or vision, numbness and tingling on one side of the body [1].

Rehabilitation can be a long process lasting over a year or it may be a relatively short couple week long program [2]. Physical and mental evaluations of the patients are usually taken regardless of the length of the program. Some examples of mental evaluation tests include the Folstcin Mini-Mental State Examination and the Geriatric Depression Screening Scale [3]. Functional and physical evaluation is usually done via the Functional Independence Measure scale (FIM). Under a FIM scale, a high number means that the patient is completely independent and a low number indicates that they need a great deal of help to function [3].

One of the most common forms of testing functional improvements for stroke patients is by utilizing a hand dynamometer. These devices can be made out of many different materials and have a variety of ranges of the force they measure. A hand dynamometer measures the grip force applied to it with a spring, hydraulic instruments, pneumatic instruments, or a strain gauge [4]. This grip strength can then be converted to a number on the FIM scale and used to determine how much care that patient should be receiving. This type of test is used to test strength after hip fractures, spinal cord injuries, certain strokes, and other injuries [5] [6].

Wrist and hand strength are related to the FIM and self-care and, as such, affect activities of daily life. The rehabilitation involved with patients who have suffered from strokes that have affected activities of daily life can be tested using a hand dynamometer [5]. The hand dynamometer will give a measure of grip strength, which when taken routinely over a period of time, will reflect on improvements or worsening conditions of the patient.

1.2 Problem Statement

In order to rehabilitate patients who have recently suffered strokes, hand dynamometers are used to measure the strength of their grip. The goal of this project is to design a dynamometer that is relatively inexpensive and that can accurately measure grip strengths of much weaker magnitude.

1.3 Current Designs

Pictured in Figure 1 are two examples of hand dynamometers. The patient reaches his or her hand across the two bars and squeezes on the one furthest from them. This displacement in the outer bar is then read by the gauge at the top as the grip strength of the individual. It is common for these dynamometers to keep a record of the maximum grip strength of the patient.

There are two major problems with this design. The first is cost. The two dynamometers shown below are priced at $330 for Figure 1.1 and $325 for Figure 1.2, and it is not uncommon to see such devices going for $500 or more. The second is accuracy-specifically, the accuracy of reading smaller values (below 20 pounds). The devices on the market now have trouble reading smaller forces with the same degree of accuracy as higher forces-this, needless to say, is a dilemma for weaker patients, such as serious stroke victims and elderly patients, who cannot deliver as much force.

Figure 1. Two commonly used hand dynamometers. Figure 1.1 shows the Jamar model, while Figure 1.2 shows the Baseline model. The Baseline model is slightly less expensive than the Jamar model. Figures from http://www.thehumansolution.com/jahady.html.

1.4 Motivation

Stroke is a leading cause of long-term disability [7]. As such, it impacts many people's lives. For the patients who have had the function of their hands or whole arm affected, it is important to have a consistent, accurate device to measure improvements in grip strength. In the Orthopedics and Rehabilitation Department of the University of Wisconsin hospital, Elizabeth Bourne and the other staff currently use hand dynamometers to test the grip strength of their rehabilitating stroke patients. They, and potentially many other stroke rehabilitation facilities, have the need for an updated and improved device to effectively aid the aforementioned patients.

An additional incentive is cost. Most of the current designs that would fit our client's requirements are several hundred dollars up to over a thousand dollars. An economic, relatively cheap design of a hand dynamometer would greatly benefit hospitals and other institutions.

2.0 Design Specifications and Client Requirements

The client had a number of specific requirements for the project, which are described in full detail below.

The device must be capable of measuring forces from 0 to 9.07 kg (0-20 lbs) in at most .454 kg (1 lb) increments. The device must be fitted with an LCD screen that will read out the exact maximum grip strength of the patient. It also must be able to be recalibrated if necessary. Materials used in the designing of the device must not include latex or sponge-like foam. Some patients may have allergies to latex, while foam is difficult to clean and has limited durability.

In addition to the above, the client presented a list of priorities ranging from highest priority to lowest. Her priorities are described further in the design matrices section. They include, in order of importance, functionality, reliability, durability, portability, and safety. The client noted that the aesthetics of the device is not important, as long as it is fully functional.

3.0 Design Alternatives

Our design process consists of two separate concepts. One set of ideas is devised to determine the best way to measure the force generated by a patient's grip, while the second is based on the identification of the best grip apparatus. Three alternatives are evaluated under the force measurement category: spring, extensometer, and strain gauge. Under the grip apparatus category, two options are included: squeeze ball and 2-Bar. All designs are described in full detail below.

3.1 Force Measurement

The force measurement alternatives are focused on how to measure the force generated by a patient's grip.

3.1.1 Spring

In the spring design, a patient pulls or pushes on a lever (see Figure 2). This action causes a spring to extend. The displacement in the spring is used to calculate the force made by the patient. This design is similar to a spring scale and is simple to use. To calculate this force, we can use Hooke's Law. This gives the equation:

$$F = -kx$$

where F is the force, k is the spring constant, and x is the displacement. The spring constant would just have to be known upon purchase of the desired spring for this design. One problem with this design is the conversion of the spring force into an electrical output so it can be displayed on the LCD screen.

Figure 2. Above is an example of a spring in use to give a force in pounds.

3.1.2 Extensometer

An extensometer is a clip-like device (see Figure 3). When the clips are pushed together, two bars are pushed apart. The distance between the two bars corresponds to a force. Extensometers measure small forces, are inexpensive, and very easy to use. Large force could make the extensometer cumbersome in its size, as it would need to be bigger to accurately reflect the correct force. Also, this design would need to have the force converted to an electrical output for the LCD screen.

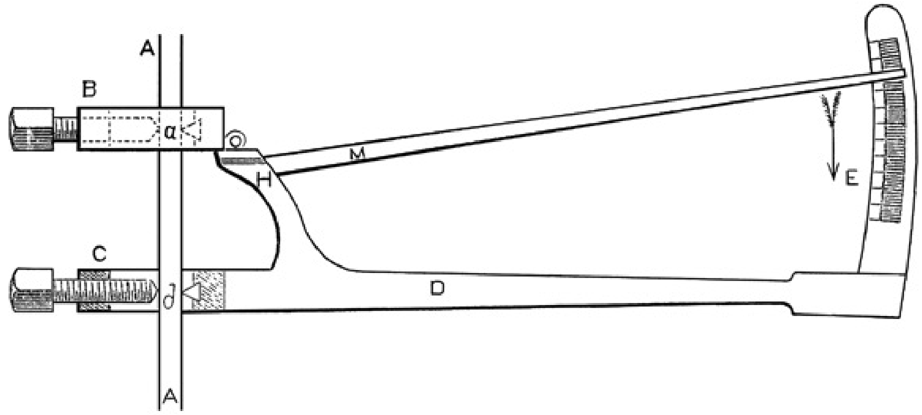

Figure 3. Above is an example of an extensometer showing the “clip” part that is pushed together.

3.1.3 Strain Gauge

A strain gauge is a small, thin piece of metal foil. It is used to measure the strain in material it is adhered to by bending with the material. Because a strain gauge is so thin, it is correspondingly able to bend and form fit with the smallest strain applied to the material. Depending on the force of the strain, a voltage is created in the strain gauge. This voltage is electrically outputted to the user. Due to the fact that it is so small and able to bend with the material, it can measure the deformation in the material it is adhered on. It is able to do this by reading a voltage drop over a Wheatstone bridge as the material deforms. Since it reads a voltage, we will be able to send this into a program to convert it to a force. This would then be inputted directly to the LCD screen.

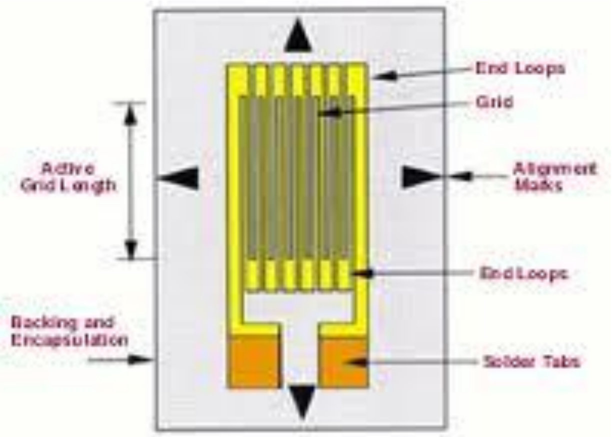

Figure 4. Above is a picture of a strain gauge that would be adhered to a metal bar.

3.2 Grip Apparatus

The grip apparatus alternatives focus on the design of the grip a patient will employ when using the device.

3.2.1 Squeeze Ball

This design encloses the measurement apparatus in a ball. A patient squeezes the bar, like a stress ball. The force of the squeeze is then measured. The exertion of the grip would be centered on the center of the squeeze ball. This would allow the pressure to push a spring or extensometer together, which would be located in the center of the ball. The strain gauge would be able to pick up the deformation in the material of the ball. The maximum amount of force might be limited in this design depending of the stiffness of the material used to make the ball. If a material that gives easy, the force being measured would have a small upper limit to it. Materials not made of latex are available to fabricate this design such as rubber.

Figure 5. Above is how the patient would grip the squeeze ball design.

3.2.2 2-Bar

This design is similar to the current device (see Figure 1). A patient pulls two bars together. One bar is curved to place the patient's hand in the same spot for every use. In this design, the two bars would be the lever to pull the spring apart to measure the force or push them together to measure the force. In the extensometer design, the two bars being pressed together. For the strain gauge design, another metal bar would have to attach the two bars. The strain gauge would then read the deformation of this connecting bar by the force the patient puts on the device by gripping the two bars together. This design would be able to use metal as the bars, which are obviously not made of latex. The grip for the bars would just have to not be latex based. Silicone is a possible example for the grip material. It is believed that this grip apparatus system would put the limiting factor of the maximum force reading of the force measurement apparatus. This is due to the fact that the two bar system is believed to be more versatile in its design.

4.0 Design Matrices

We generated two design matrices to evaluate the main design factors, force measurement and grip apparatus, separately. The matrices have identical categories. Category weights are assigned and distributed by our client according to her preference and opinions regarding the degrees of importance of the different categories.

The following categories were identified: functionality, reliability, durability, portability, and safety. Functionality was defined to be the ability of the design to give the maximum grip readout if we had fabricated the design ourselves. Reliability was defined as being consistent and give the correct force put on the device. Durability was defined as its ability to withstand cleaning and the force being put on the device when in use.

Our client chose functionality and reliability as the most important part of the designs. It is important that the device be consistent and function accurately every time. The next most important category was durability. The device needs to be able to withstand cleaning after each use since it is to be used in a hospital. The device also needs to be portable. The least important category was safety. Although safety is an important component of any design, the safety of this project is very limited in scope. The design is only dangerous if the device contains latex, as some patients could be allergic. It should also be noted that appearance and aesthetics of the design is not important to the client.

4.1 Force Measurement Design Matrix

After rating each mechanism in each category a final score was tabulated. As shown in Table 1, the strain gauge was chosen for the force measurement. Functionality and reliability were the main factors that separated strain gauge from spring and extensometer. This was because a strain gauge already generates an electrical output of the force generated. In the spring and extensometer designs, we would have to generate this output ourselves, which could pose a problem and increase the difficulty of the design.

Table 1: Force Measurement Design Matrix. Design matrix comparing the possible alternatives of the measurement of force. Categories were selected and weighted by client. The scores shown were equated using a range from 1 to 5 and then multiplied by the category weight.

Category (weight)	Spring	Extensometer	Strain Gauge
Functionality (6)	24	24	30
Reliability (6)	24	24	30
Durability (4)	15	5	15
Portability (3)	15	15	15
Safety (1)	5	5	5

Total (100)	83	73	95

4.2 Grip Apparatus Design Matrix

The grip apparatus was put through the same matrix, with the same categories. As shown in Table 2, the 2-bar design outperformed the squeeze ball. Again, functionality and reliability were the deciding factors. In the squeeze ball design, a ball of the proper hand dimensions for max grip is not big enough to encompass the whole hand. If the ball's size is increased, the palm of the hand is not used in the grip, which is not what our client wants. The 2-Bar design incorporates the whole hand in concert with the proper grip of a hand.

Table 2: Grip Apparatus Design Matrix. Design matrix comparing the possible alternatives of the different grip designs. Categories were selected and weighted by client. The scores shown were equated using a range from 1 to 5 and then multiplied by the category weight.

Category (weight)	Squeeze Ball	2-Bar
Functionality (6)	18	30
Reliability (6)	18	30
Durability (4)	15	15
Portability (3)	15	15
Safety (1)	5	5
Total (100)	71	95

5.0 Final Design

The chosen final design combines the traditional, two bar grip of existing dynamometers with the efficiency and accuracy of strain gauges. The grip force can then be measured and output as an electrical signal. This eliminates the need to convert from a mechanical spring force or distance measurement to an electrical signal. The signal will be output to an external instrumentation unit that houses all of the electrical components including the LCD screen, battery, circuit elements, and button. The grip and instrumentation unit is connected by a flexible, non-latex rubber wire cover. The entire product has all electrical components sealed off so that it can be sanitized when needed. The cost of all materials can be found in the appendix.

5.1 Grip Apparatus

The grip is designed as a familiar, two bar system fabricated from aluminum. This offers comfort while maximizing grip force. 3.80 (cm) ** is the optimal grip diameter when measuring maximum grip force. This is an average measurement from a study conducted on a variety of male and female adults, all of different ages. The bar that is placed against the palm is slightly wider than the bar gripped by the fingers. This feature improves comfort and enables the user to fully grasp the bar. Each handle is wrapped in self-adhering silicon tape to provide stability when gripped and minimize errors or damage from slipping. This is an important feature because the patients using this product are weaker and could possibly drop the grip, so this will reduce that risk.

The handle that is placed against the palm has a small hand stop protruding from the top. Patients will be instructed to place the top of their hand firmly against the stopper, and then wrap their fingers around the full grip apparatus. This will increase the accuracy of the force measurement, as well as consistency between visits. The two handles are connected by a small aluminum piece that have the strain gauges mounted to it.

5.2 Strain gauge

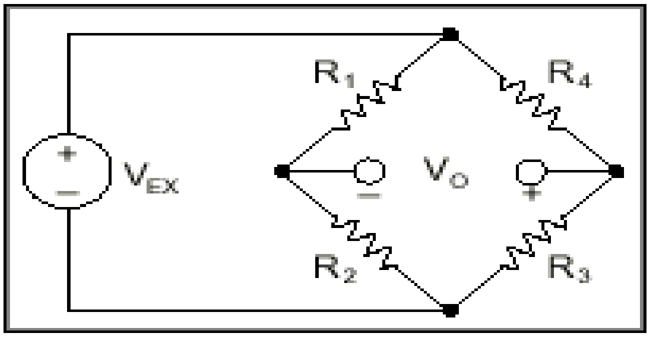

Figure 6: Wheatstone Bridge (http://zone.ni.com/devzone/cda/tut/p/id/10636)

Strain gauges are a multi-functional class of sensors that convert physical phenomena, such as tension, compression, or pressure, into readable electrical signals. They are bonded directly to the surface of the body in which strain is induced. The signal is produced by connecting the strain gauges in a special circuit configuration known as a Wheatstone bridge (Figure 4). In this specific implementation of the Wheatstone bridge, resistors R_3 and R_4 are strain gauges. R_3 is bonded to the bottom of the aluminum connector, while R_4 is bonded to the top. When the handle is gripped, R_4 will be in tension and the resistance will increase, while R_3 will be in compression and resistance will decrease. This creates a potential difference across V_o that can be amplified and related directly to strain. Further calculations can be seen in the appendix.

a

b

The strain gauge has two wire leads that are soldered to two terminal pads which are bonded to the finger handle. 28 gauge instrumentation wire is then bonded to these pads and ran to the instrumentation unit. The purpose of the terminal pads is to eliminate any external strain induced by the instrumentation wire pulling on the strain gauge. All of these components will be carefully sealed with a non-corrosive, non-conductive silicone caulk.

5.3 Instrumentation Unit

The instrumentation unit is connected to the grip apparatus by a long flexible rubber wire cover as mentioned above. It is the center for all electrical components. Four lead wires (two for each strain gauge) are run from the housing to the grip through the wire cover. The rest of the half-bridge circuit is found inside of the unit. This external entity will hold the visible LED screen, power switch, battery, instrumentation amplifier, filter circuitry, button, and the microcontroller. It is sealed using the silicone caulk mentioned above. This protects the electrical components from any water damage and allows for easy sanitation. It will be fabricated using the 3d printer.

5.4 Microcontroller

The brain of the instrumentation unit is the microcontroller. For this application, an mbed NXP LPC1768 with an ARM Cortex-M3 Core processor will be used [4]. This microcontroller allows sensitive, machine-level control, while providing a user-friendly online compiler complete with a wide variety of predefined libraries. C++ is the language utilized by the mbed software.

The program is fairly simple. It runs on a loop triggered by the button. When the device is powered on, the LCD displays instructions to press the button to begin testing. It then gives a 5 second countdown so the grip can be properly situated in the patients' hand, and then it begins taking readings. Readings are taken every tenth of a second for five seconds and each one is displayed continuously on the LCD screen. The maximum grip is stored and displayed after the five second grip is finished. The user then presses the button again to take another reading.

6.0 Testing

In order to test the effectiveness of the strain gages in measuring applied forces, a strain gauge was attached to an aluminum metal bar. One end was secured to a table and the aluminum was positioned so that the other end hung over the edge. Then, weights were hung from the loose end, observing the value displayed by the LCD screen and comparing it to the actual measured weight. This was done for weights ranging from 0.87 to 20.87 pounds, with a single 0-pound measurement on each trial.

This test was performed a total of nine times. For each trial, the value on the equation was changed slightly, and the changes in the total values were observed and recorded. The nine trials provided R^2 values ranging from .9964 to .9985, indicating a very high level of accuracy between actual weight and displayed force. The value of the R^2 was not a perfect one due to the amplifier not being a good fit for this purpose. This is explained more in future work. In the end, a single value was chosen for the value that reflected the greatest level of accuracy among the trials we performed. Trail results can be seen in the appendix.

7.0 Future Work

The mbed is a powerful prototyping tool; however it is too powerful for this application. The mbed itself consumes around .150 Amps, dramatically reducing battery life for the product. This is due to other features constantly being active. One solution to this problem is to implement, in the program, functions that turn off these unneeded components. The code for this is complex and would take a long time to properly incorporate without hindering the mbeds overall performance. A longer term solution that is feasible on a mass-production scale is to design a PCB that utilizes the ARM Cortex-M3 processor, but does not include the mbed itself. All code written for the mbed will still work, but power consumption would be greatly reduced.

The LPC1920 instrumentation amplifier currently used in the circuit is unsatisfactory for this application. It is a dual-supply, high voltage amplifier that has the ability to be used as a single-supply, low voltage amp, but does so with greater error. It was used because it came highly recommended due to ease of use and versatile functionality. However, it was discovered that when used single-supply, there were fairly strict requirements to get proper readings, and there was much greater error. This amplifier can be replaced easily with a low-voltage, single supply instrumentation amplifier that will better suit this application and increase accuracy.

The client specified that they wanted the ability to recalibrate the device. The chosen solution is to have the device zero itself every time it is powered on. This function would have inaccurately zeroed the device if implemented with the improper instrumentation amplifier, however it will be easy to program with the proper amplifier in place.

One improvement that can be made is with the silicone tape grip. Instead of this tape, a silicone dip can be used. This would give a more uniform distribution of the silicone as well as additional safety to the user by covering up sharp edges. Another improvement is to design a containment case for the strain gauge to help protect it. One problem that was found when trying to design one is the case cannot come in contact with the staining gauge, the bar it is adhered to, and the wires coming from the strain gauge.

The next steps in the process are to finish connecting the containment box to the device after the box is finished being printed. The device will then be delivered to the client and tested in the hospital to see how it works in a real world setting.

8.0 References

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2. "Post-Stroke Rehabilitation Fact Sheet", NINDS. October 2008. NIH Publication No. 08-4846

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9.0 Appendix

9.1 PDS

Product Design Specification

Problem Statement

Stroke is one of the leading causes of adult disability in the United States. It is caused by the disturbance of blood supply to the brain leading to loss in brain functions. Stroke can lead a person to occupational disability, wherein the person is unable to perform the functions required to complete their daily functional tasks satisfactorily. In the Orthopedics and Rehabilitation Department at the UW hospitals nurses and doctors treat patients with physical, occupational and speech therapy. They assess the recovery of upper extremity function in stroke patients with occupational disability, using a grip meter or dynamometer. The grip meter measures the grip strength of an individual.

Current grip meters available in the market are expensive and do not allow measurement of forces due to grip from 0 to 20 lb with 1 lb increment, limiting the ability to measure small changes in very weak patients.

The project consists of designing an ergonomically suitable grip meter that will allow measurement of 1 lb force with a range from 0 to 20 lb and necessary calibration before use. The grip meter should also provide a digital readout of the force on an LCD screen.

Client Requirements:

· Must have at least 1 lb increments

-Must have a max grip read out on a LCD screen

-Must at least measure from 0 to 20 lbs

-Must have the ability to be recalibrated if necessary

-Must have no latex

Design Requirements:

1. **Physical and Operational Characteristics**

a*. Performance Requirements*

The grip meter must be able to display the max grip on an LCD screen. The device must be portable and hand-held.

b. *Safety*

The device must not contain any latex, as it is to be used in a hospital setting. Water/liquids should not be used around the device as this could pose a threat of electrical shock to the user. Cleaning agents are safe to use on the grip apparatus part of the device. No sharp corner in the device should exist.

c. *Accuracy and Reliability*

Our design must at least have 0.454 kg (1 lb) increments and be able to read from a range of 0 kg (0 lbs) to 9.072 kg (20 lbs). It also must be able to be recalibrated to make sure it remains accurate in the future.

d. *Life in Service / Shelf Life*

The device is believed to be used around 5 to 10 times a month. The shelf life will depend on the circuits remaining intact and the strain gauge remaining plastic. The client expects the device to last three years.

e. *Operating Environment*:

Our device would be used in a hospital setting, but could be used in any environment. The device should not be used in or around water. Water will damage the device and could harm user. Other liquids pose a similar risk.

f. *Ergonomics*:

The device must be extremely user friendly. Anyone should easily be able to use the device. The therapist should be able to hold the LCD screen containment box, while the patient is using the device. Wiring should not be in the way of the patient or therapist. The device was made to have the therapist facing the patient so the wiring from the grip apparatus to the box does not affect either person.

g. *Size*:

The device should not be too cumbersome in its length. If the device bars are too long, patients could have a problem using and holding onto the device. Currently, the device bars are 14 cm (5.5 in) long. The client has stated that this will not be an issue at this length.

h. *Weight*:

The device must be lightweight so a patient recovering from a stroke will be able to lift and hold onto it. A believed maximum weight for the device is 2.268 kg (5 lbs). This weight constriction was given to us by the client. Currently, the device weighs .

i. *Materials*:

The client has requested that the materials be with able to withstand cleaning agents and no made of latex. The materials must also be durable and last for at least three years before breaking. The materials used were aluminum and silicon. These materials meet all requirements given by the client for materials.

j. *Aesthetics, Appearance, and Finish*:
It should be noted that the appearance of the device does not matter to the client. The device is to be covered with silicon tape as a grip, and the containment box for the circuit board and LCD screen is made out of a plastic polymer. These materials were used since they did not contain latex.

2. Production Characteristics

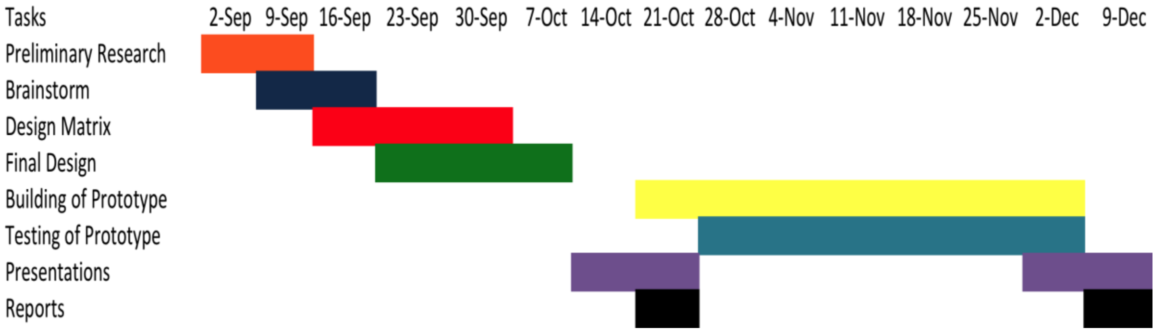
a. *Quantity*: 1 deliverable.
b. *Target Product Cost*: $310

3. Miscellaneous

a. *Standards and Specifications*: N/A
b. *Customer/Patient related concerns*: N/A
c. *Competition*: There are other grip meters currently on the market. The ones typically used in a rehabilitation setting can cost around $500.

9.2 Timeline

Below is the timeline for the semester.



9.3 Budget

Below is a list of materials used and the corresponding cost.

Strain gauge material	$102
Mbed	$75
Aluminum	$68
Silicon tape/rubber caps	$20
Circuitry	$30
Battery	$14
Other	$7

Total $316

9.4 Testing Data

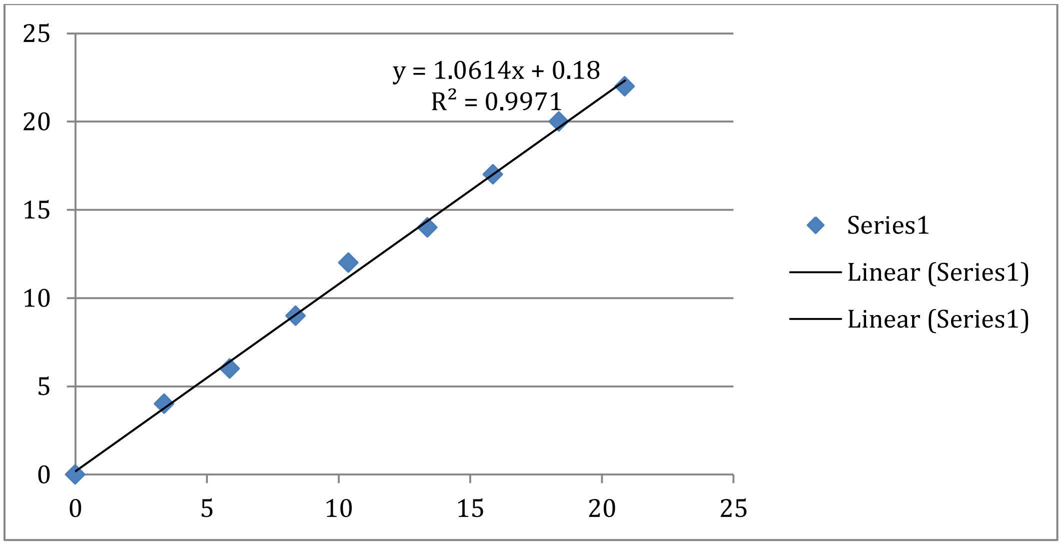


Figure 7. Above is the testing data for trial 1.

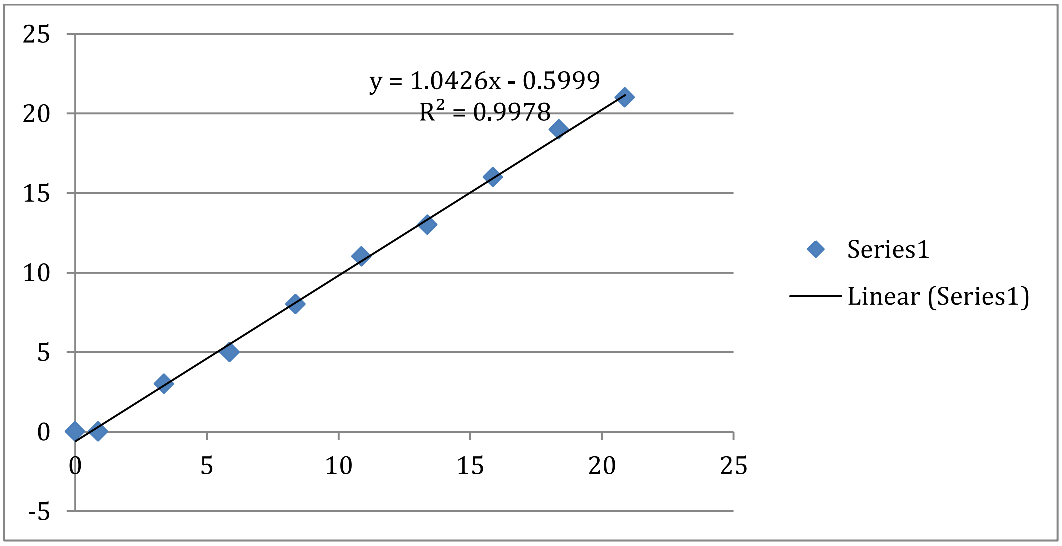


Figure 8. Above is the testing data for trial 2.

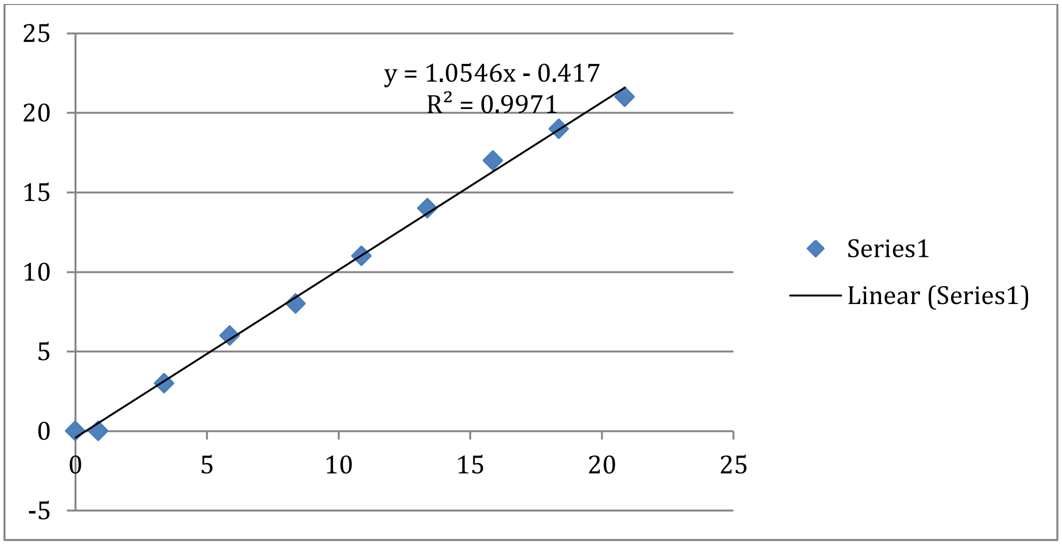


Figure 9. Above is the testing data for trial 3.

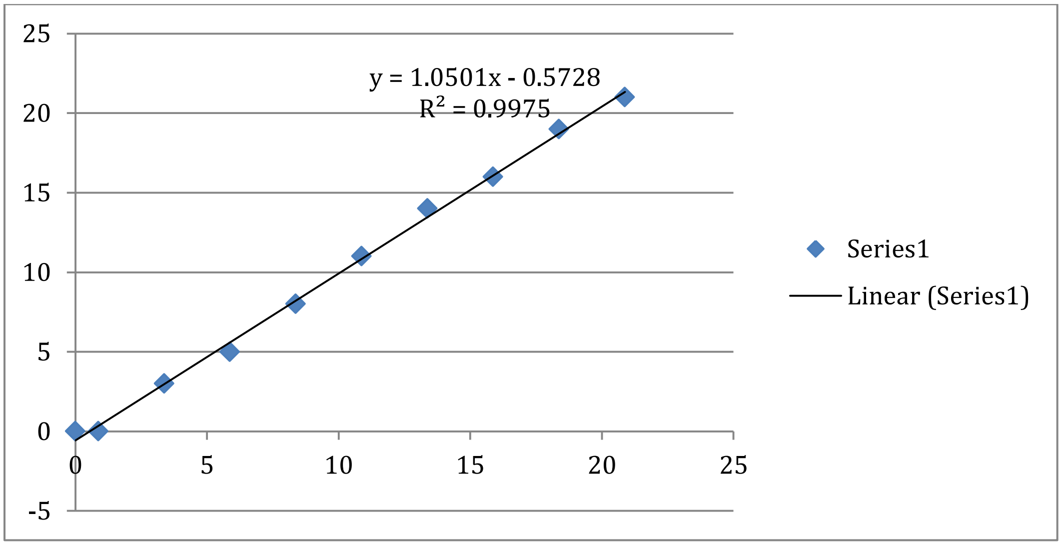


Figure 10. Above is the testing data for trial 4.

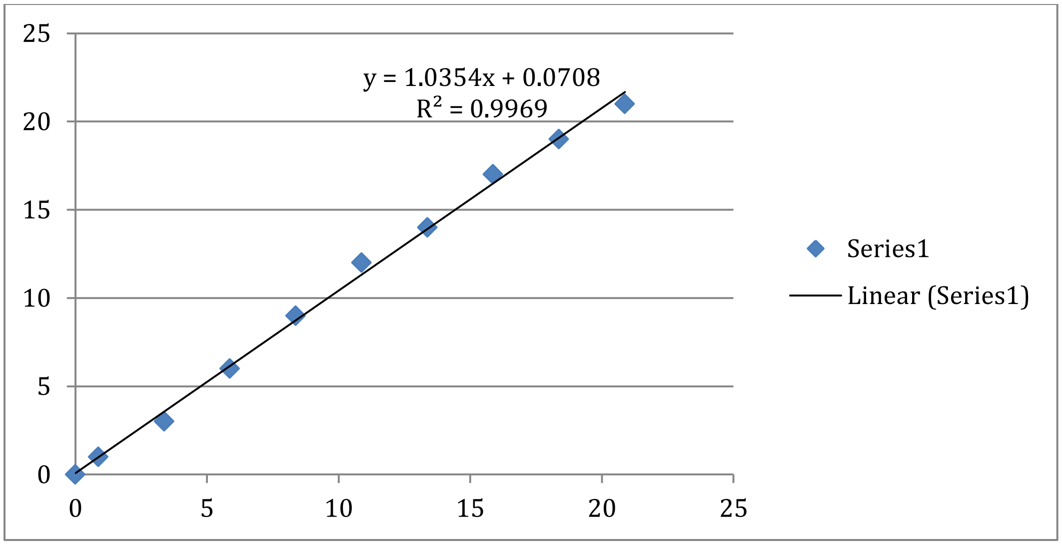


Figure 11. Above is the testing data for trial 5.

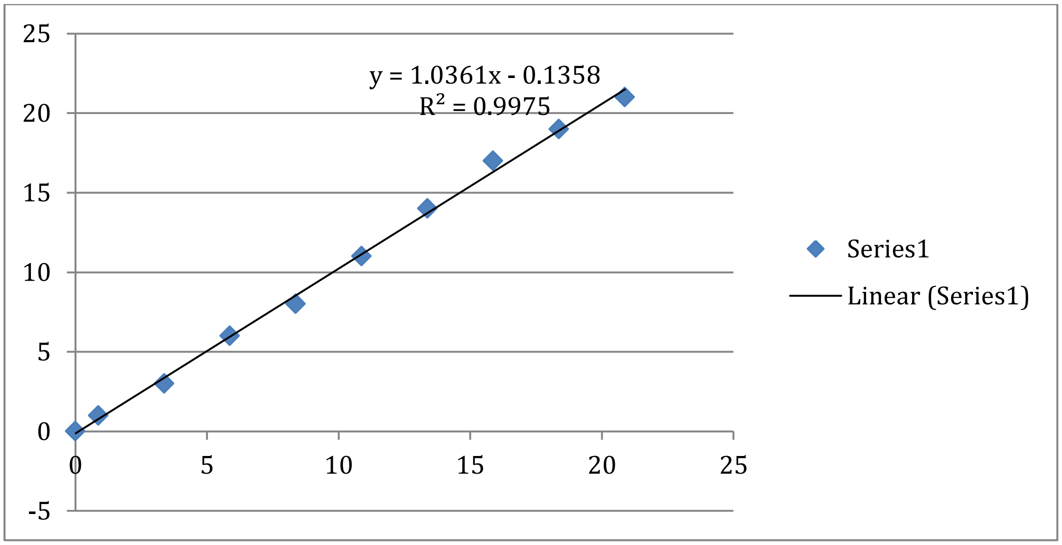


Figure 12. Above is the testing data for trial 6.

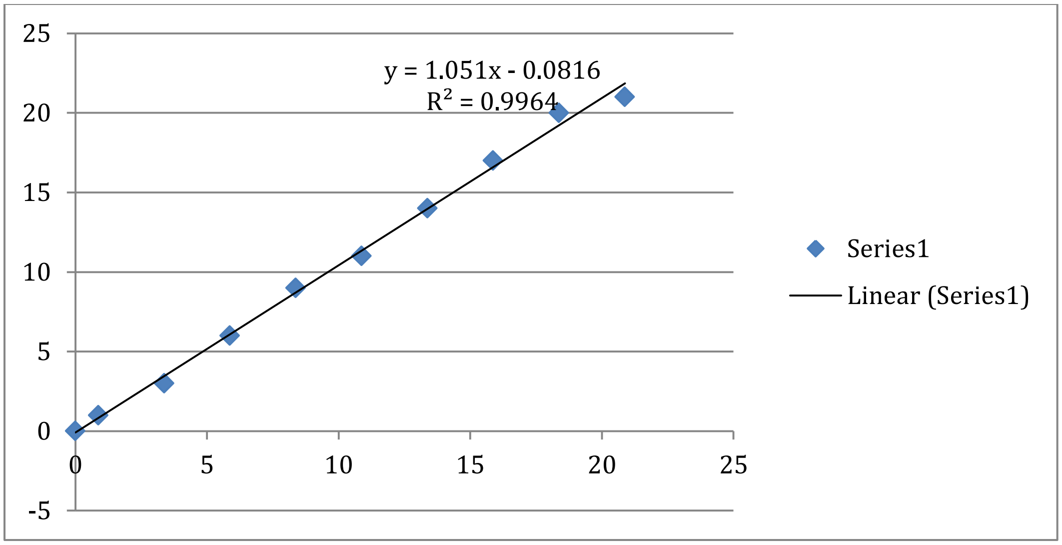


Figure 13. Above is the testing data for trial 7.

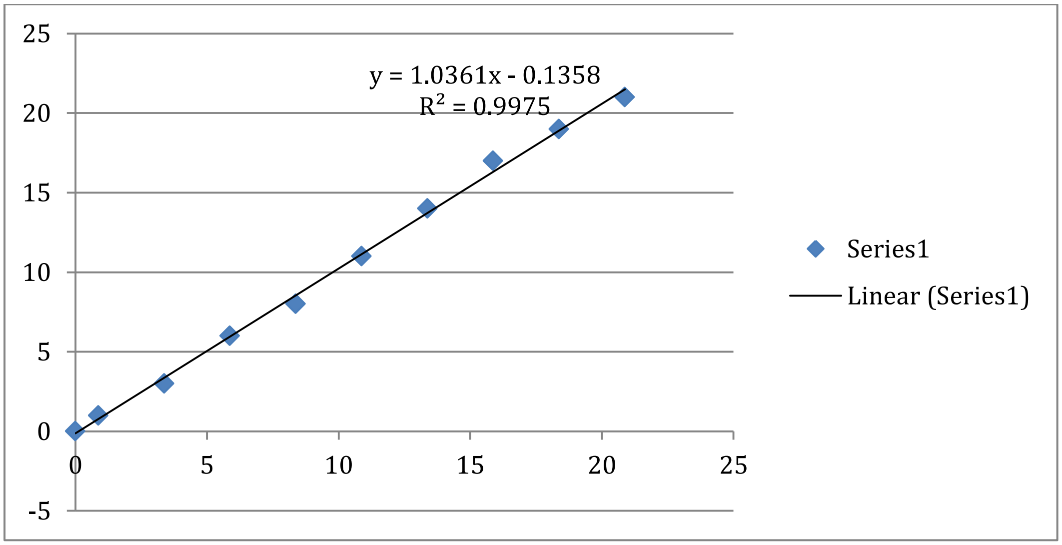


Figure 14. Above is the testing data for trial 8.

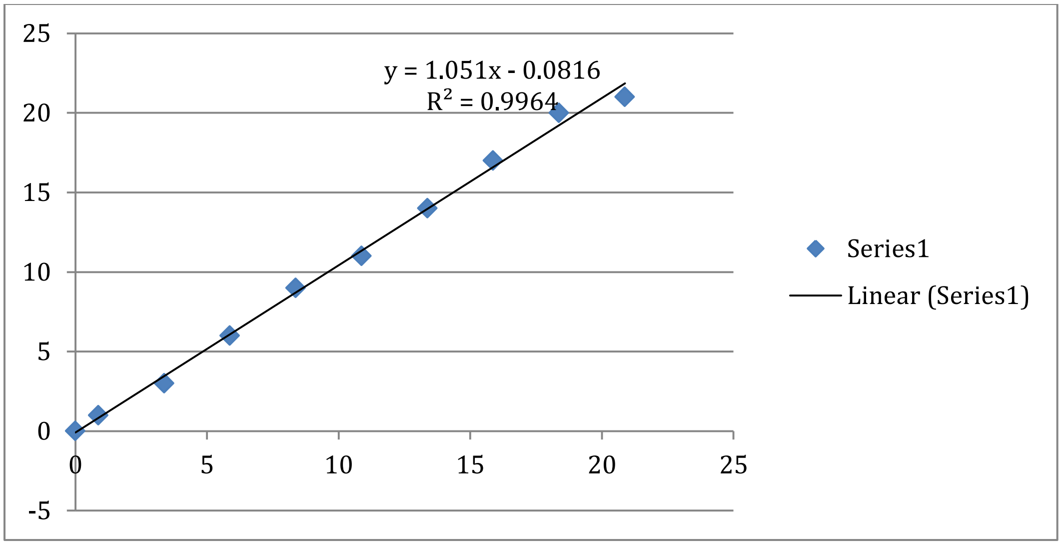


Figure 15. Above is the testing data for trial 9.

8.5 Calculations

Change in resistance (ΔR) divided by the original resistance (R) is linearly related to strain by the gauge factor:

$$(1)\qquad \frac{\Delta R}{\mathrm{R}} = \kappa * \varepsilon$$

Substituting in for ε and rearranging:

$$(2)\quad \kappa = \frac{\frac{\Delta R}{\mathrm{R}}}{\frac{\Delta \mathrm{L}}{\mathrm{L}}}$$

$$(3)\quad \Delta R = \kappa * \varepsilon * R$$

Using circuit analysis techniques, the voltage at nodes a (V_a) and b (V_b) can be found, as well as $V_{o.}$

$$(4)\quad Va = \frac{R2}{R1+R2} * Vin$$

$$(5)\quad Vb = \frac{R3}{R3+R4} * Vin$$

$$(6)\quad Vo = Vb - Va$$

In the half-bridge circuit, R_4 is assumed to be in tension, while R_3 is assumed to be in compression. R is taken as the pre-strain resistance of both strain gauges. $R_2 = R_1$ is assumed also. This gives:

$$(7)\quad R4 = R + \Delta R$$

$$(8)\quad R3 = R - \Delta R$$

Plugging V_a, V_b, R_4, and R_3 into equation (6), dividing both sides by V_{in}, and understanding V_b = ½ :

$$(9) \quad \frac{Vo}{Vin} = \frac{R-\Delta R}{(R-\Delta R)+(R+\Delta R)} - \frac{1}{2}$$

Substitute ΔR from equation (3) into equation (9):

$$(10) \quad \frac{Vo}{Vin} = \frac{R - R*\kappa*\varepsilon}{(R - R*\kappa*\varepsilon) + (R + R*\kappa*\varepsilon)} - \frac{1}{2}$$

Simplification of equation (10) gives an equation with the only unknown being strain (ε):

$$(11) \quad \frac{Vo}{Vin} = \frac{-\kappa*\varepsilon}{2}$$

5.2.2 Derivation of Force Treating the Metal Connector as a Spring

The force of a spring can be found using the simple equation:

$$(12) \qquad F = C * \Delta x$$

C is the spring constant for the metal connector (it is normally represented as K or K_s in practice but to avoid confusion with the gauge factor it has been labeled C). Δx is the displacement of the spring from the unloaded position. C, for a rigid body, can be modeled by:

$$(13) \qquad C = \frac{A*E}{Ls}$$

Where A is the cross-sectional area of the connector, E is a known material constant, and L_s is the length of the connector (Prof. Witt, Robert. Engineering Mechanics and Astronautics, UW –Madison). Once C is determined, equation (12) may be rearranged:

$$(14) \qquad \Delta x = \frac{F}{C}$$

Noticing that Δx = ΔL, since the bonded strain gauge is displaced along the connector, equation (14) may be substituted into the definition of strain:

$$(15) \qquad \varepsilon = \frac{\frac{F}{C}}{L} = \frac{F}{C*L}$$

Equation (15) may then be substituted into equation (11):

$$(16) \qquad \frac{Vo}{Vin} = \frac{-\kappa*\frac{F}{C*L}}{2}$$

Solving for F produces:

$$(17) \qquad F = -\frac{2*Vo*C*L}{\kappa*Vin}$$

This equation finds the force of the grip acting on the strain gauge, with the bending moment theoretically being accounted for by the circuitry.